HEAT TRANSFER IN A "PSEUDOTURBULENT" BED OF DISPERSED MATERIAL

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A two-phase model is proposed for the steady heat exchange between a surface and a pseudoturbulent bed of dispersed material. Expressions are obtained for the temperature fields of the gaseous and solid phases.

Processes of heat exchange in heterogeneous systems can be intensified through a transition to the so-called pseudoturbulent mode of motion of the dispersed material. Therefore, the investigation of heat conduction and heat exchange in the pseudoturbulent mode of motion of a dispersed heat-transfer agent has great practical importance. This term "pseudoturbulent transfer" presumes a certain analogy between pulsation transfer in continuous and dispersed media, although it is known that there is no physical similarity between turbulent and pseudoturbulent transfer, since the nature of the pulsation motion in the two cases is entirely different. The pseudoturbulent mode of motion of a dispersed material is observed with pulsations of moderate velocity in apparatus containing moving, fluidized, and vibrationally fluidized beds, in devices containing mixers, etc.

Heat exchange with a surface submerged in a mixed dispersed medium is usually calculated from empirical equations of the type

$$q = \alpha_W (\vartheta_W - \vartheta_\infty).$$

The drawbacks of such an approach are well known. Chief of them is the difficulty of determining the heat flux under conditions different from those under which the empirical equations for α_W were obtained. The designing of each new apparatus containing a dispersed bed requires preliminary experimental investigations. Such an empirical course is also unpromising because the value of ϑ_∞ remains indeterminate to one extent or another, since the temperature profile far from the heat source is unknown.

Various models of heat transfer in a dispersed (basically a fluidized) bed have been discussed before in [1-5]. The main drawback of the proposed models consisted in the use of the hard-to-determine time of stay of particles near the heat-exchange surface [1-4] or of the contact thermal resistance at the boundary [5] in them. Moreover, in these models it is assumed that the temperature field is localized near the heat-exchange surface, whereas in fact even a well mixed core of a bed can limit the heat flux from the wall. Such an approach can be justified in the case when the pseudoturbulent mixing is so well developed that the thermal resistance of the core in the bed can be neglected. However, to quantitatively estimate the conditions under which such a situation can set in one must first describe the total thermal resistance at the boundary and in the core of the bed. In particular, the following equation is widely used to calculate the effective thermal conductivity in the phenomenological theory of heat transfer in a turbulent stream:

$$\lambda_{ef} = \lambda_m + \lambda_s.$$

An equation of this kind can evidently also be used to describe the temperature field in a pseudoturbulent bed. At a sufficient distance from the heat-exchange boundary, neglecting the filtration component of transfer for simplicity, one can assume that the pseudoturbulent pulsation transfer of heat by particles plays the main role in the formation of the temperature field. We assume that near the surface heat exchange occurs predominantly as a result of the interaction of the temperature field formed from the gaseous phase with the tempera-

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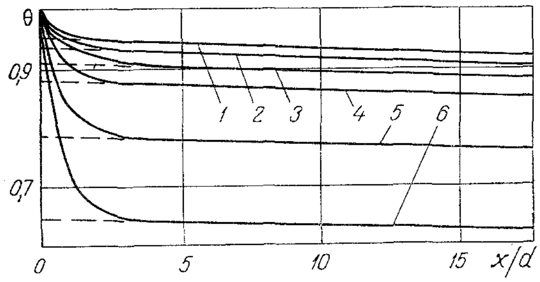

Fig. 1. Temperature distribution in a "pseudoturbulent" bed. Dashed lines) solid phase; solid lines) gaseous phase: 1) $A_h = 2$, $\lambda_g/\lambda_s = 0.02$; 2) $\sqrt{2}$ and 0.02, respectively; 3) 1 and 0.02; 4) $\sqrt{2}$ and 0.01; 5) $\sqrt{2}$ and 0.005; 6) $\sqrt{2}$ and 0.0025.

ture field of the particles. Then the model of steady pseudoturbulent transfer in a dispersed bed can be written mathematically in the form

$$\lambda_g \frac{d^2\vartheta_g}{dx^2} - \alpha^* S(\vartheta_g - \vartheta_p) = 0, \tag{1}$$

$$\lambda_s \frac{d^2\vartheta_p}{dx^2} + \alpha^* S(\vartheta_g - \vartheta_p) = 0. \tag{2}$$

Equation (1) characterizes the process of heat conduction of the gaseous phase and (2) characterizes that of the mixed solid phase.

The system (1)-(2) will be solved with the following boundary conditions:

$$\vartheta_g|_{x=0} = \vartheta_w; \quad \vartheta_g|_{x=l} = 0, \tag{3}$$

$$\left.\frac{d\vartheta_p}{dx}\right|_{x=0} = 0; \quad \vartheta_p|_{x=l} = 0. \tag{4}$$

We write Eqs. (1)-(4) in the dimensionless form

$$\frac{d^2\theta_g}{dY^2} - A_h^2(\theta_g - \theta_p) = 0, \tag{5}$$

$$\frac{d^2\theta_p}{dY^2} + A_h^2 \frac{\lambda_g}{\lambda_s}(\theta_g - \theta_p) = 0, \tag{6}$$

$$\theta_g|_{Y=0} = 1; \quad \theta_g|_{Y=L} = 0, \tag{7}$$

$$\left.\frac{d\theta_p}{dY}\right|_{Y=0} = 0; \quad \theta_p|_{Y=L} = 0. \tag{8}$$

The parameter A_h was obtained earlier in [6]. It was shown that in a dense bed the coefficient of interphase heat exchange $A_h = \sqrt{6(1-\varepsilon)Nu^*}$ displays the properties of a universal constant and is numerically equal to two. Evidently, there is no reason to assume that in pseudoturbulent beds of different types the value of the Nusselt number $Nu^* = \alpha^* d/\lambda_{ef.g}$ should undergo any significant change. We note that the quantity A_h will change for pseudoturbulent systems of different types, since it depends on the porosity ε of the bed.

From Eq. (5) we express θ_p through θ_g:

$$\theta_p = -\frac{1}{A_h^2}\frac{d^2\theta_g}{dY^2} + \theta_g. \tag{9}$$

Substituting (9) into (6), we obtain

$$\frac{d^4\theta_g}{dY^4} - A_h^2\left(1 + \frac{\lambda_g}{\lambda_s}\right)\frac{d^2\theta_g}{dY^2} = 0. \tag{10}$$

The solution of Eq. (10) has the form

$$\theta_g = C_1 + C_2 Y + C_3 \exp\sqrt{\beta}\,Y + C_4 \exp(-\sqrt{\beta}\,Y). \tag{11}$$

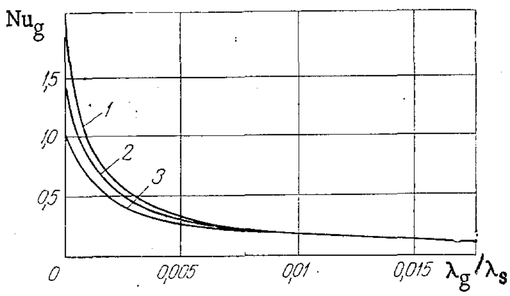

Fig. 2. Dependence of dimensionless heat flux Nu_g at the wall on the ratio λ_g/λ_s: 1) $A_h = 2$; 2) $\sqrt{2}$; 3) 1.

Substituting (11) into (9), it is easy to obtain an expression for the temperature of the solid phase,

$$\theta_g = C_1 + C_2 Y - C_3 \frac{\lambda_g}{\lambda_s} \exp \sqrt{\beta} Y - C_4 \frac{\lambda_g}{\lambda_s} \exp(-\sqrt{\beta} Y), \tag{12}$$

where

$$\beta = A_h^2 \left(1 + \frac{\lambda_g}{\lambda_s}\right).$$

The constants C_1, C_2, C_3, and C_4 appearing in Eqs. (11)-(12) are found from the boundary conditions (7) and (8). By substituting (11) into (7) and (12) into (8) it is easy to obtain

$$C_4 = 1 + \frac{1}{\sqrt{\beta} L} \frac{\lambda_s}{\lambda_g} - \frac{1}{\exp(-2\sqrt{\beta} L)} + \frac{1}{\sqrt{\beta} L \exp(-2\sqrt{\beta} L)} \frac{\lambda_s}{\lambda_g}, \tag{13}$$

$$C_3 = -C_4 \exp(-2\sqrt{\beta} L), \tag{14}$$

$$C_2 = \sqrt{\beta} \frac{\lambda_g}{\lambda_s} C_3 - \sqrt{\beta} \frac{\lambda_g}{\lambda_s} C_4, \tag{15}$$

$$C_1 = 1 - C_3 - C_4. \tag{16}$$

Let us determine the value of the dimensionless heat flux at the wall:

$$Nu_g = -\left.\frac{\partial \theta_g}{\partial Y}\right|_{Y=0}. \tag{17}$$

Substituting (11) into (17) and using (13)-(15), after simple transformations we obtain

$$\begin{aligned} Nu_g = {} & A_h \left(1 + \frac{\lambda_g}{\lambda_s}\right) \sqrt{1 + \frac{\lambda_g}{\lambda_s}} \left[1 + \exp\left(-2A_h \sqrt{1 + \frac{\lambda_g}{\lambda_s}}\, L\right)\right] \times \\ & \times \left[1 + A_h \frac{\lambda_g}{\lambda_s} L \sqrt{1 + \frac{\lambda_g}{\lambda_s}} - \exp\left(-2A_h \sqrt{1 + \frac{\lambda_g}{\lambda_s}}\right) + \right. \\ & \left. + A_h L \exp\left(-2A_h L \sqrt{1 + \frac{\lambda_g}{\lambda_s}}\right) \sqrt{1 + \frac{\lambda_g}{\lambda_s}}\right]^{-1}. \end{aligned} \tag{18}$$

Let us analyze Eq. (18). As $\lambda_s \to \infty$ (ideal mixing) the maximum value of Nu in a pseudoturbulent bed is equal to A_h. A similar result was obtained earlier for nonsteady heat exchange between a surface and a bed of stationary dispersed material [7], as well as for heat exchange between a surface and a bed of mixed dispersed material [5]. The dependences calculated from Eqs. (11), (12), and (22) with different values of the parameters A_h and λ_g/λ_s are presented in Figs. 1 and 2.

From Fig. 1, in which the temperature distributions are presented, it is seen that the temperatures of the gas and the solid phase are practically equalized at a distance of several particle diameters from the wall, i.e., interphase heat exchange between the gas and particles is localized near the heat-exchange surface. The length of the section of temperature equalization depends on the parameter A_h. For the same ratio λ_g/λ_s this section is the larger, the smaller A_h.

From the dependence of the dimensionless heat flux Nu_g at the wall on λ_g/λ_s presented in Fig. 2 it is seen that heat exchange with the surface intensifies with an increase in λ_s. As $\lambda_s \to \infty$ the value of Nu approaches its maximum value.

In the authors' opinion, the proposed model can be useful in the generalization of experimental data on heat exchange in pseudoturbulent moving, fluidized, vibrationally fluidized, and other beds of dispersed material. But the proposed approach cannot yet be considered as the only possible one for the description of temperature fields in the continuous and dispersed phases of a pseudoturbulent bed.

NOTATION

λ_g, effective thermal conductivity of gaseous phase; λ_s, effective thermal conductivity of the mixed solid phase; ε, porosity; λ_m, molecular thermal conductivity; d, particle diameter; ϑ_∞, temperature of dispersed bed at a large distance from heat source; ϑ_g, gas temperature; ϑ_p, particle temperature; ϑ_w, wall temperature; x, current coordinate in the direction perpendicular to the wall; l, bed thickness; q, heat flux; α, coefficient of heat exchange between wall and pseudoturbulent bed of dispersed material; α^*, coefficient of interphase heat exchange; $\theta_g = \vartheta_g/\vartheta_w$, dimensionless gas temperature; $\theta_p = \vartheta_p/\vartheta_w$, dimensionless particle temperature; $Y = x/d$, dimensionless coordinate; $L = l/d$, dimensionless bed thickness; A_h, dimensionless coefficient of interphase heat exchange; $Nu_g = \alpha d/\lambda_s$, Nusselt number.

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